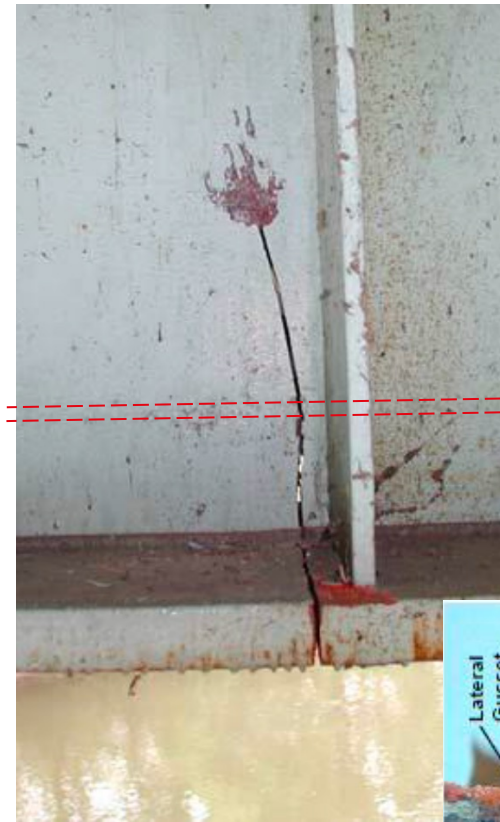


Part 3: Introduction to Fracture Mechanics (ref. TGC 10 § 13.3)

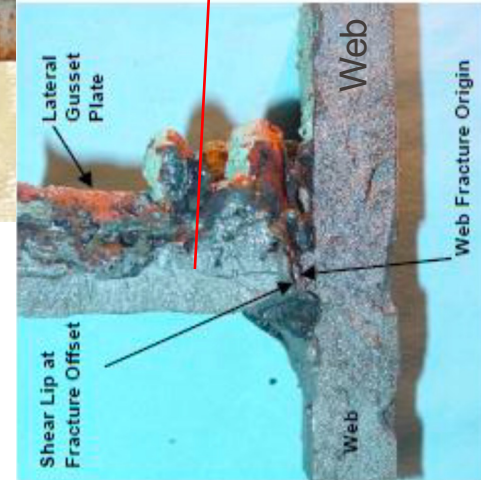
Prof. A. Nussbaumer
RESSLab

CIVIL526-Steel structures,
selected chapters

- Introduction
- Stress Intensity Factor
- Influence of shape, correction factors
- Other influences:
 - Environment
 - Similarity conditions (deformation state, 2D vs 3D)
- Toughness $\Leftrightarrow$ Charpy V
(Ténacité $\Leftrightarrow$ Résilience)

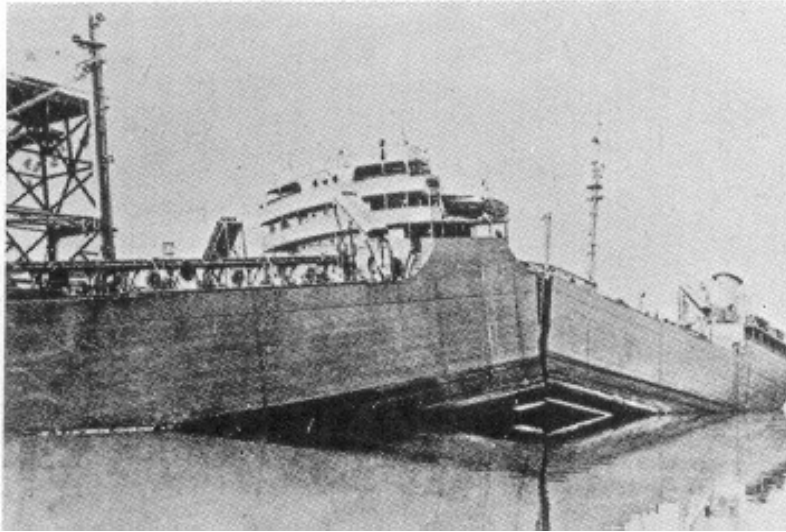
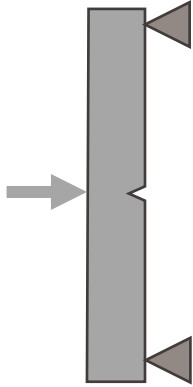


Longitudinal
Stiffener



Kaufmann, and al. (October 2004). Failure Analysis of the US 422 Girder Fracture. *National ATLSS center, Lehigh Univ.*

Charpy test (impact)



Fracture of the USS Schenectady "liberty ship" in San Diego Harbor

⇒ poor welds, high clamping/triaxiality, and low temperature

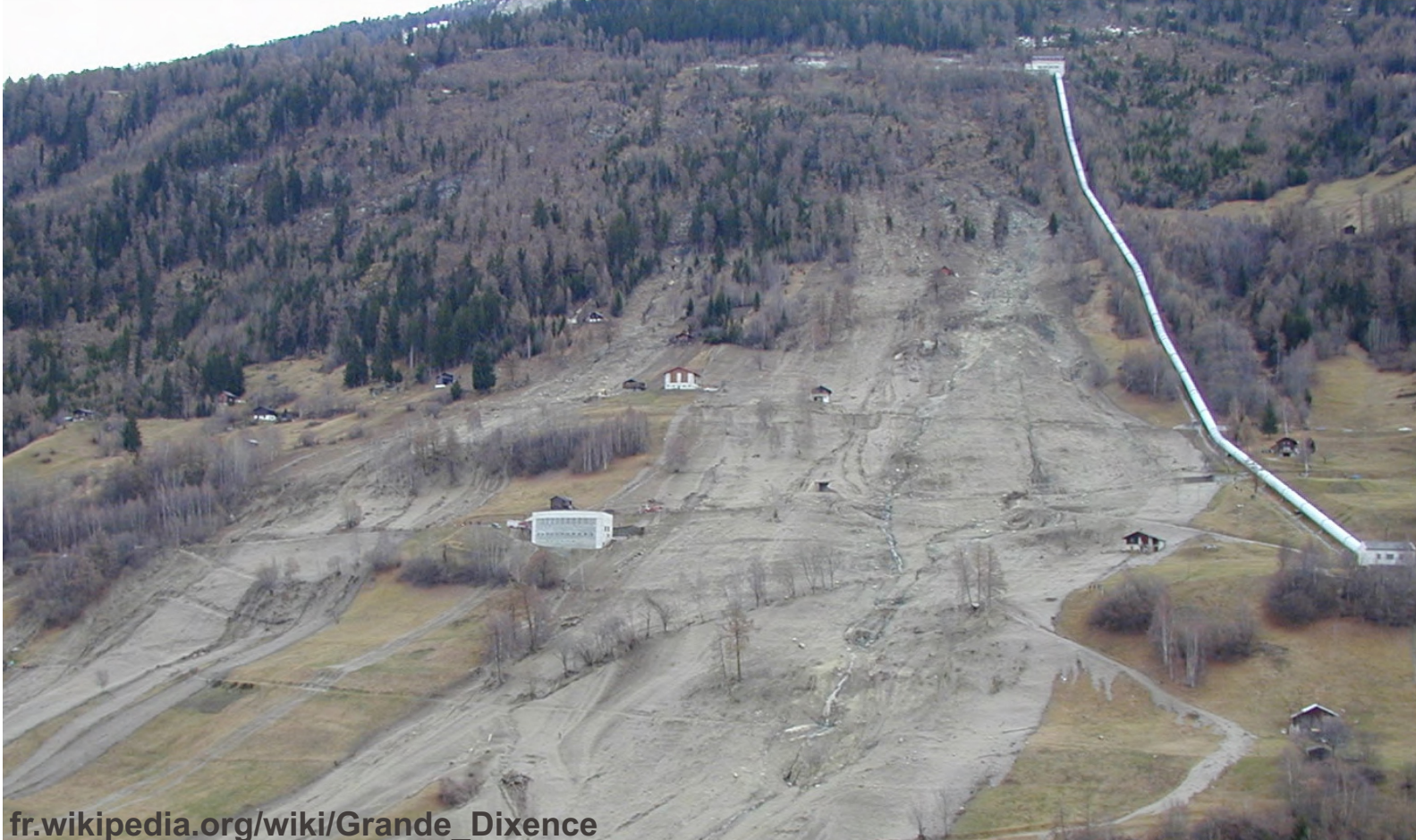
⇒ again in 1979, same type of failure:
MSV Kurdistan oil tanker

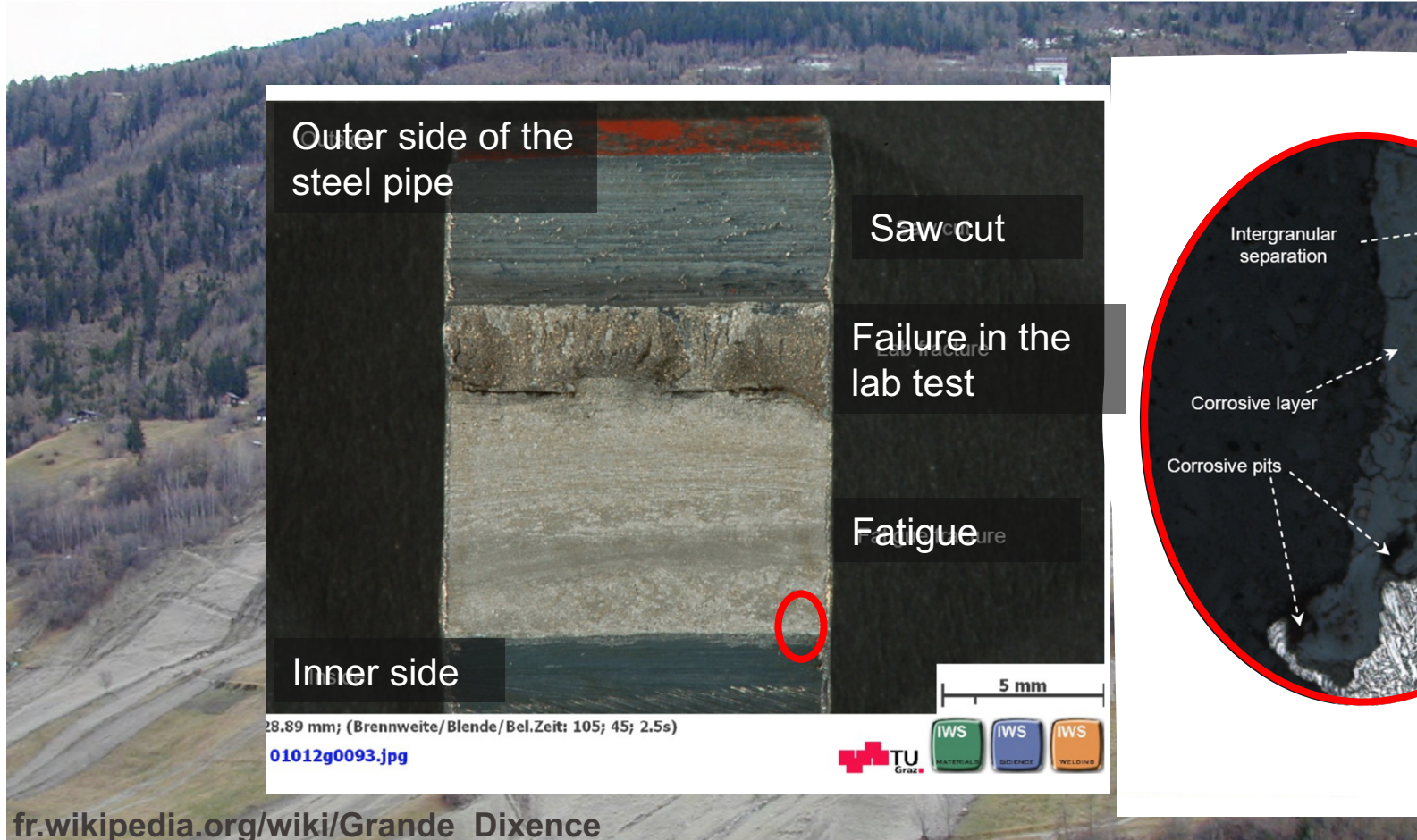


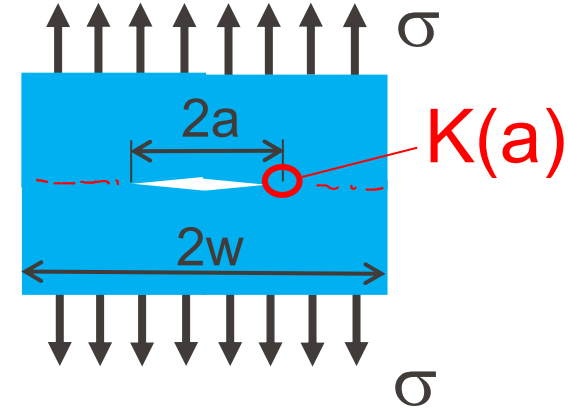
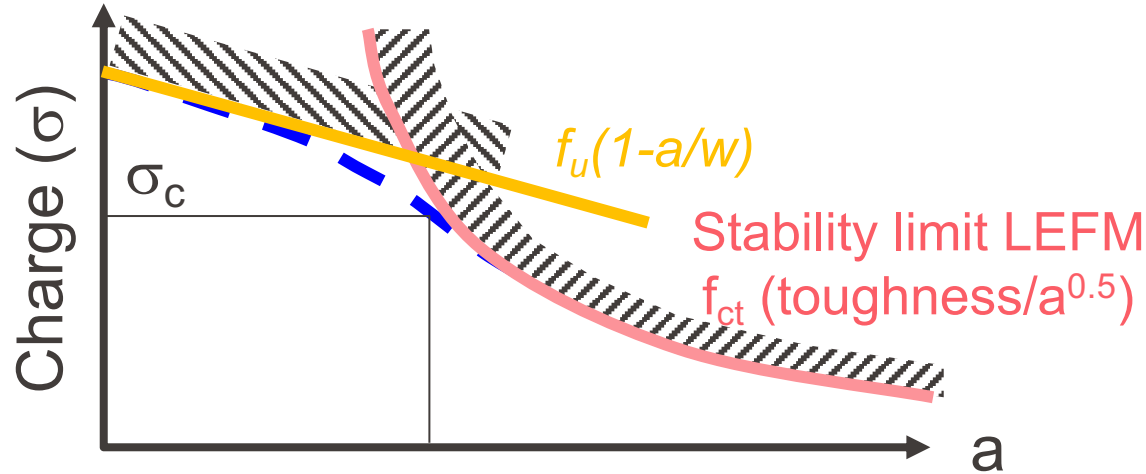
G.R. Irwin (left) and J.A. Kies

Failure problems solved in the 1950s:

- Failure of Liberty ships
- High-altitude explosions of DeHaviland Comet aircraft
- Explosions of electrical generators
- Explosions of combustion chambers in HSS for rockets.





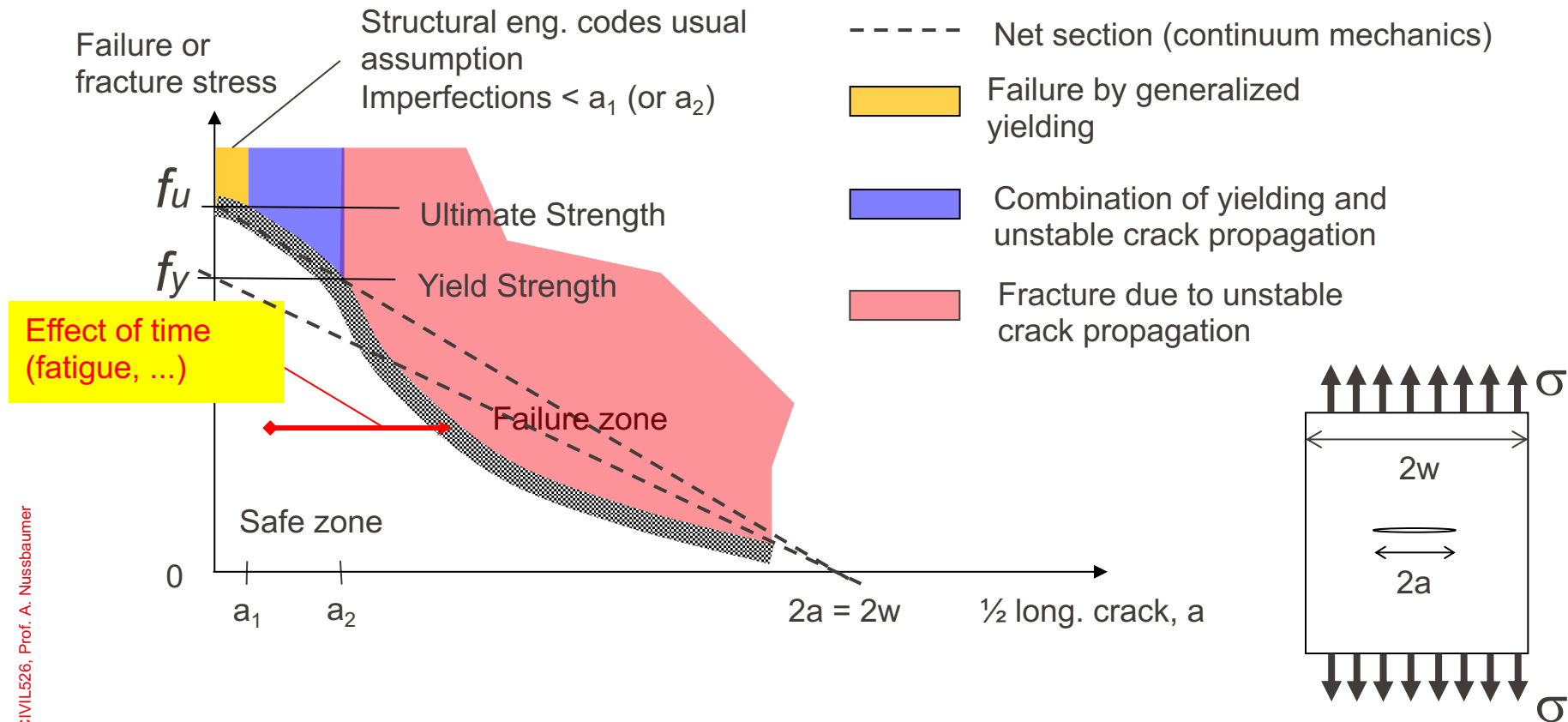


Parameters:

- Imperfections, concentration σ
- Effects of scale, triaxiality
- Residual stresses

EPFL INTRO Tolerance or residual strength in the presence of a crack ⁷

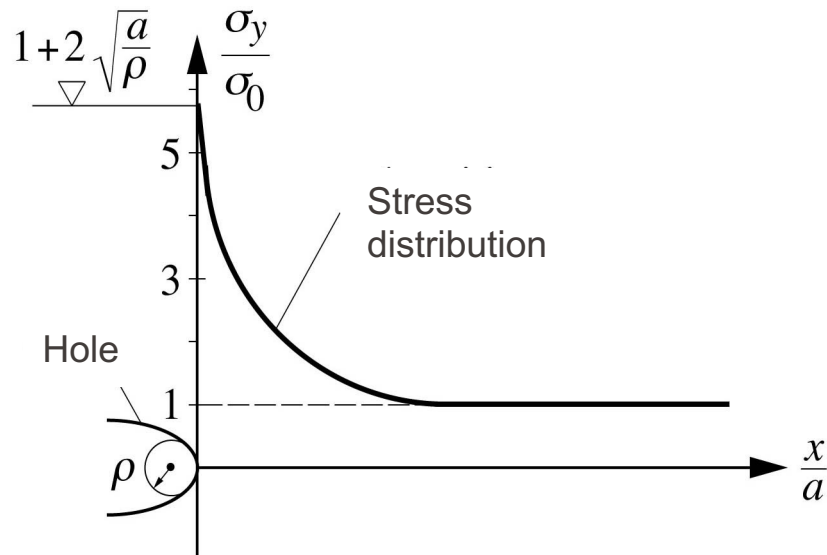
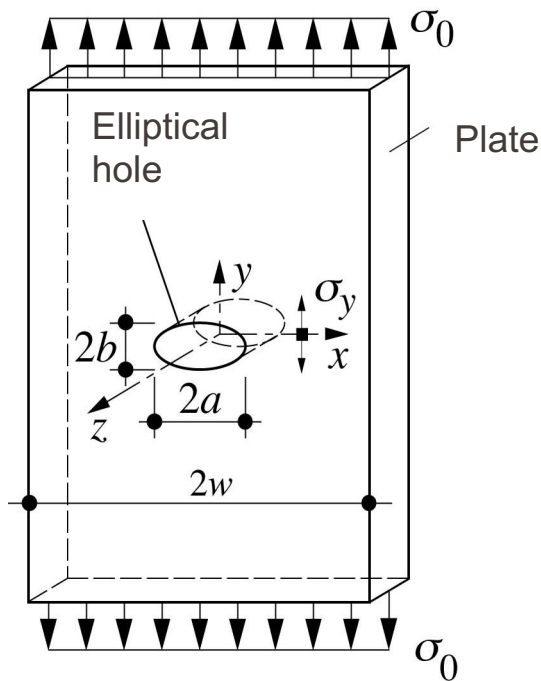
At a given temperature



INTRO reminder: basic problem of non-continuum mechanics (at fracture)

Resolved by Timoshenko:

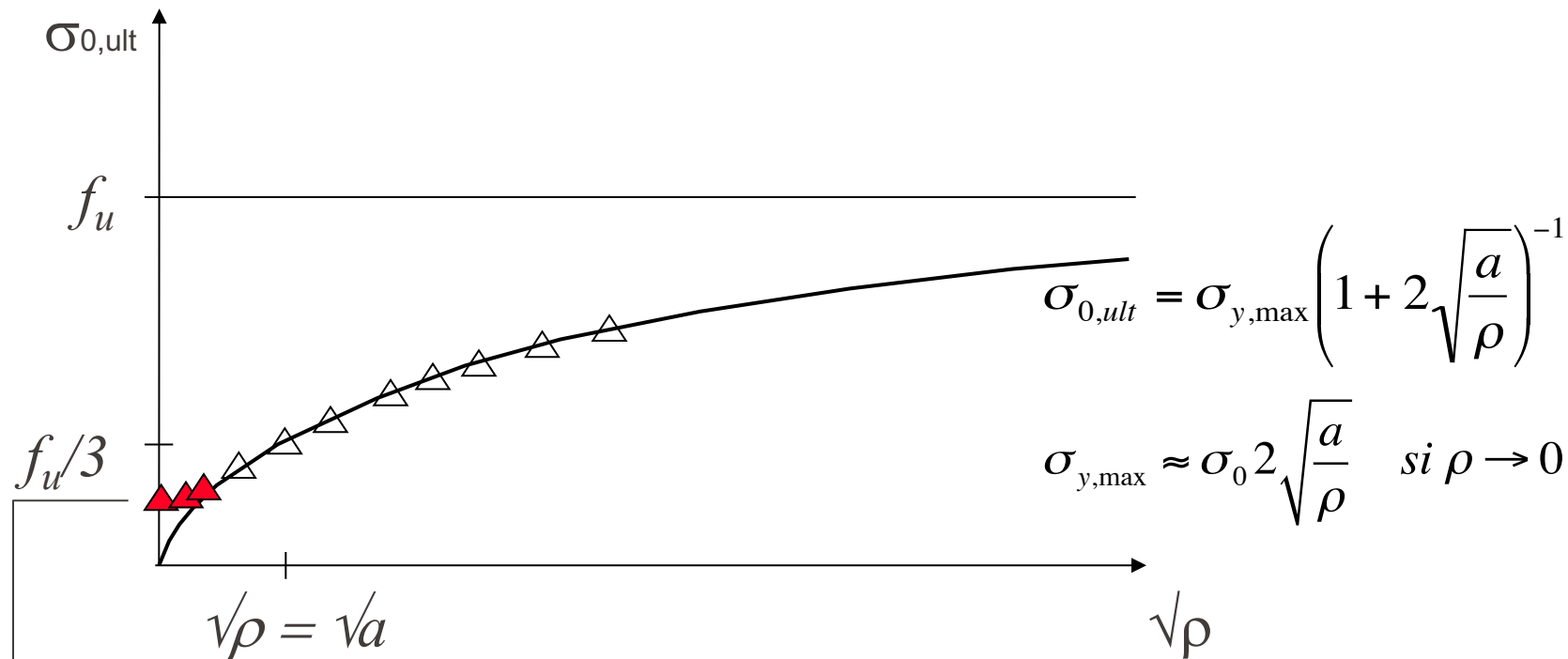
$$\sigma_{y,\max} = \sigma_0 \cdot k_t = \sigma_0 \left(1 + 2 \frac{a}{b} \right) = \sigma_0 \left(1 + 2 \sqrt{\frac{a}{\rho}} \right)$$



(TGC 10, 2015, fig. 13.4)

Important note: even if drawn in 3D, the problem and its solution are 2D

INTRO: basic problem of non-continuum mechanics (at fracture)

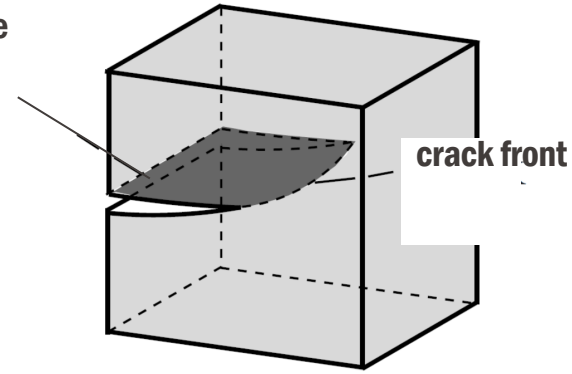
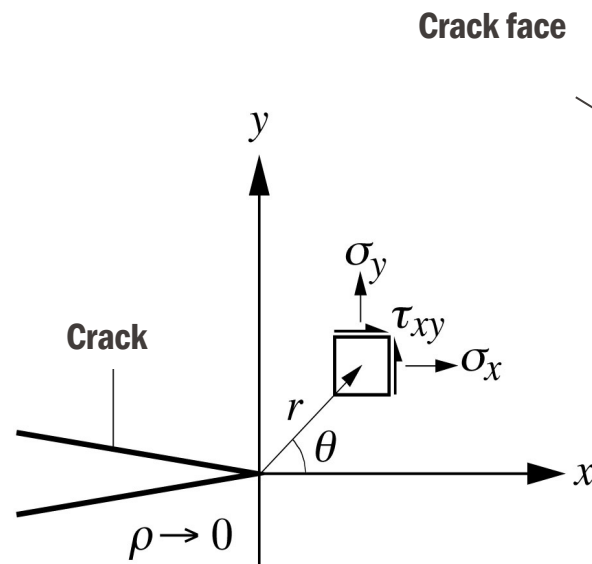
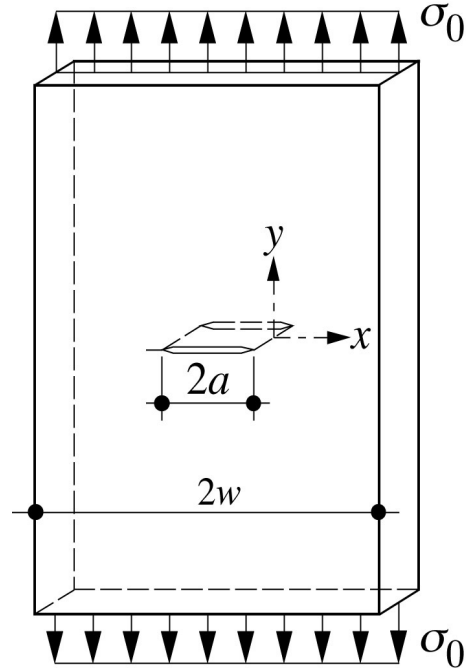


$\sigma_{0,ult} \sqrt{a} = \lim_{\rho \rightarrow 0} \frac{1}{2} \sigma_{y,max} \sqrt{\rho} \rightarrow \text{Toughness of material}$

Content

- Introduction
- **Stress Intensity Factor**
- Influence of shape, correction factors
- Other influences:
 - Environment
 - Similarity conditions (deformation state, 2D vs 3D)
- Toughness $\Leftrightarrow$ Charpy V

Fig. 13.5: Stress field near a crack



$K = ?$ With the only variables of the problem:

$$\sigma_{ij} = \frac{K_I}{\sqrt{2\pi r}} f_{ij}(\theta)$$

$$K_I \approx \sigma_0 \sqrt{a}$$

Verification: $K_{Ed} \leq K_{Rd}$ (toughness)

$$\sigma_x = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left(1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right)$$

$$\sigma_y = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left(1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right)$$

$$\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}} \sin \frac{\theta}{2} \cos \frac{\theta}{2} \cos \frac{3\theta}{2}$$

$$K_I = Y \cdot \sigma_0 \cdot \sqrt{\pi a}$$

$$Y = Y_e \cdot Y_f \cdot Y_s$$

shape finite dim. surface

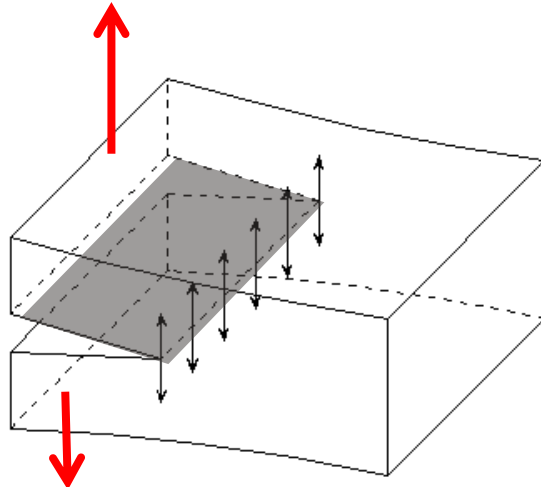
K_I [MPa·mm^{1/2}]: Stress Intensity Factor

Y [-]: Geometrical correction factor

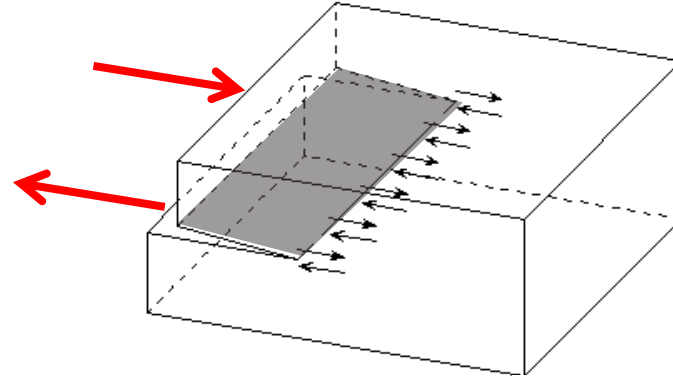
σ_0 [MPa]: Normal stress perp. to the crack

a [mm]: Crack dimension (depth or half-length)

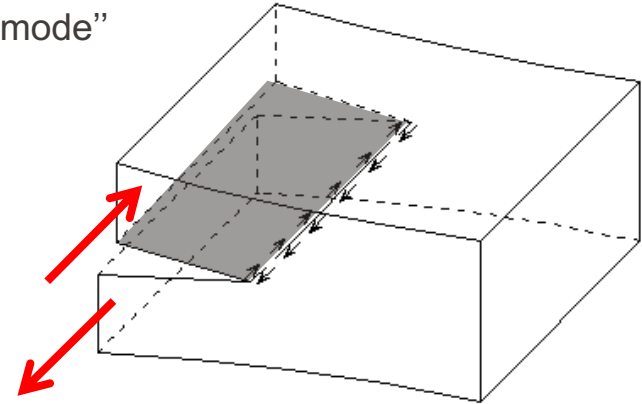
The 3 crack faces displacement modes



(a) Mode I "opening mode"



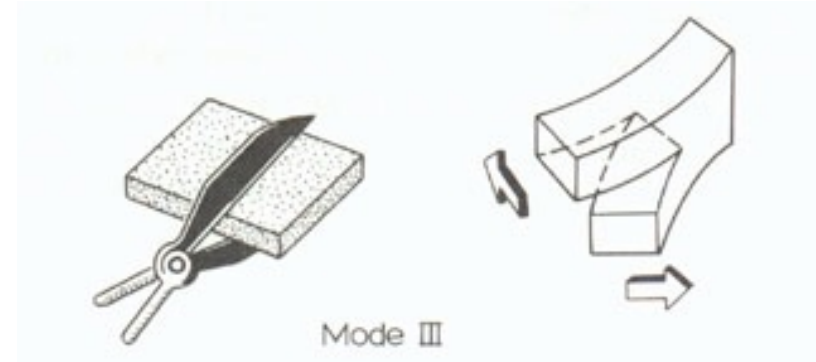
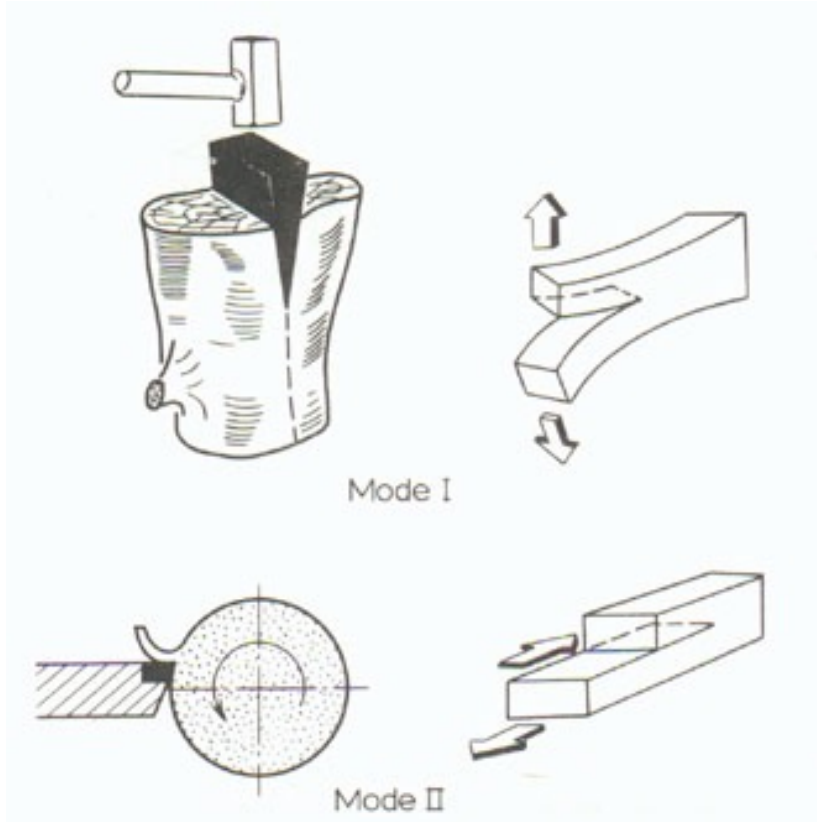
(b) Mode II "shearing mode"



(c) Mode III "tearing mode"

Important note: even if drawn in 3D,
the problem and its solution are 2D

Practical examples of the 3 Modes



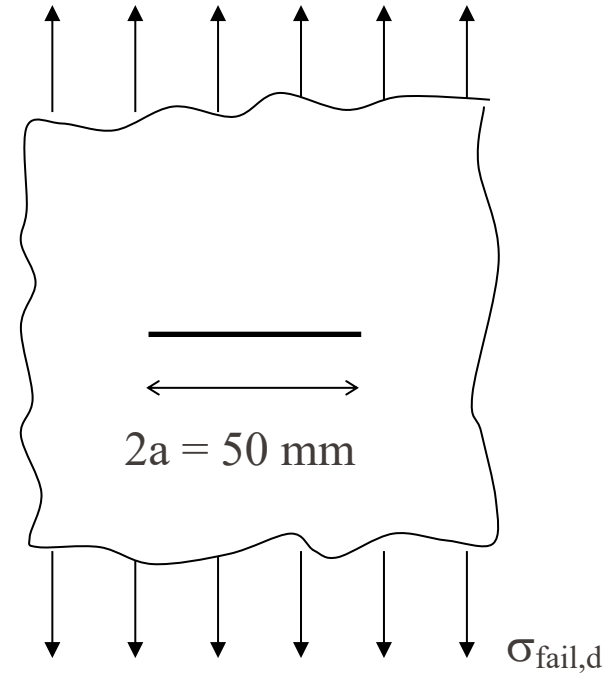
(From Parton V.Z. *Fracture Mechanics from Theory to Practice* Pg. 66 Figure 47, Gordon and Breach Science Publishers.)

EPFL Simple application example

15

- Aluminium alloy plate
(7075, T651):
 - $f_u = 560 \text{ N/mm}^2$
 - $E = 70 \text{ kN/mm}^2$
 - Toughness $K_{Ic} = 920 \text{ N/mm}^{3/2}$
 - $\gamma_{safety} = 1.5$

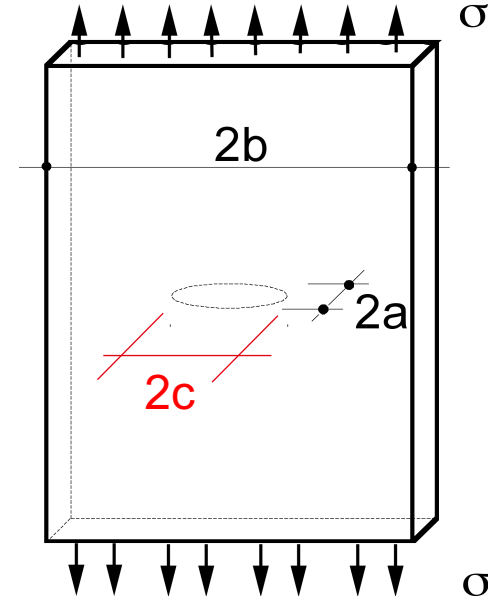
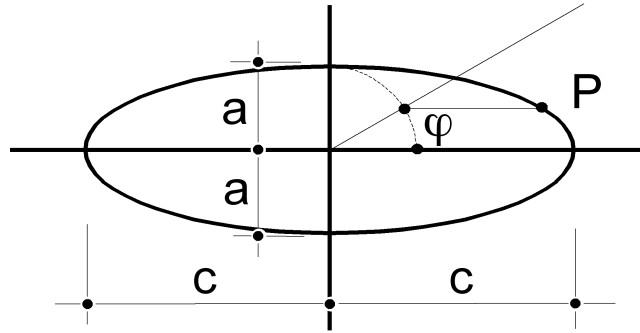
$$\sigma_{fail,d} = ?$$



Content

- Introduction
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- Toughness $\Leftrightarrow$ Charpy V

Solutions for K: an elliptical crack in a horizontal plane loaded perpendicularly (approx. by Irwin)



$$K_{I,P} = \frac{\sigma \sqrt{\pi a}}{\phi} \left(\sin^2 \varphi + \frac{a^2}{c^2} \cos^2 \varphi \right)^{1/4}$$

$$\phi = \sqrt{1 + 1,464 \left(\frac{a}{c} \right)^{1,65}} \quad \text{pour} \quad \frac{a}{c} \leq 1$$

Generic equations for elliptical cracks

- Newman & Raju solution (NASA, 1970's):

$$Y = F \cdot f_w \cdot (F_{km} F_m + F_{kb} F_b)$$

<https://ntrs.nasa.gov/archive/nasa/casi.ntrs.nasa.gov/19840015857.pdf>

F_i , f_w influence : 1, 1/2 ou 1/4 ellipse, finite dimensions, loading mode (tension or bending)

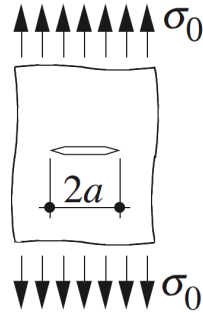
- TGC 10, only solutions in tension, but different cracks (edge, semi-elliptical, ...):

$$Y = Y_e \cdot Y_f \cdot Y_s$$

Fig. 13.6: K for different geometries

Bidimensionnal case

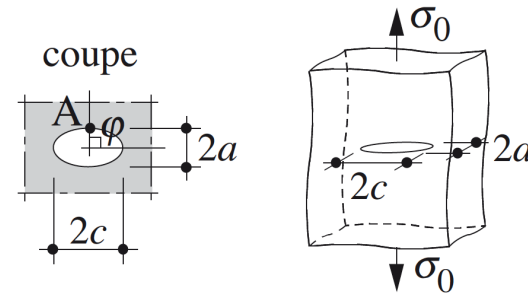
Infinite plate



$$K = \sigma_0 \sqrt{\pi a}$$

Tridimensionnal case

Correction for the crack's shape



For point A ($\varphi = \pi/2$):

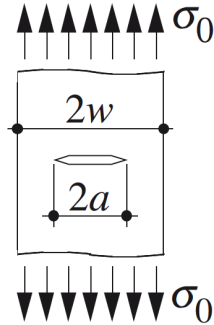
$$Y_e = \frac{1}{\int_0^{\pi/2} \left[1 - \frac{c^2 - a^2}{c^2} \sin^2 \theta \right]^{1/2} d\theta}$$

Approx.:

$$Y_e = \left[1 + 1.464 \left(\frac{a}{c} \right)^{1.65} \right]^{-1/2} \quad \frac{a}{c} \leq 1$$

Fig. 13.6: K for different geometries

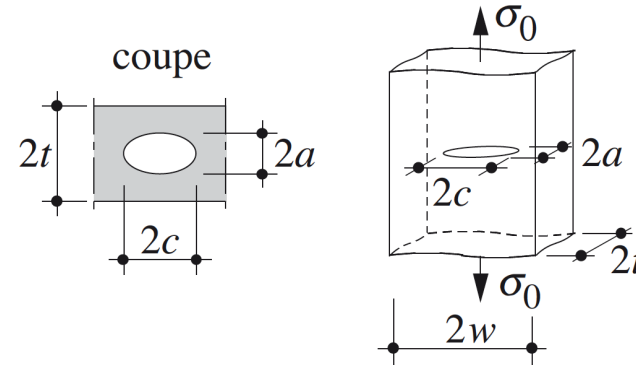
Correction for the plate
dimension



$$Y_f = \frac{1}{\sqrt{\cos \frac{\pi a}{2w}}}$$



Correction for the plate
dimensions

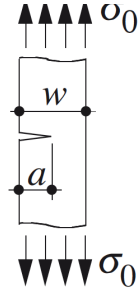


$$Y_{fa} = \sqrt{\frac{2t}{\pi a} \tan \frac{\pi a}{2t}} \quad \text{et} \quad Y_{fc} = \sqrt{\frac{2w}{\pi c} \tan \frac{\pi c}{2w}}$$



Fig. 13.6: K for different geometries

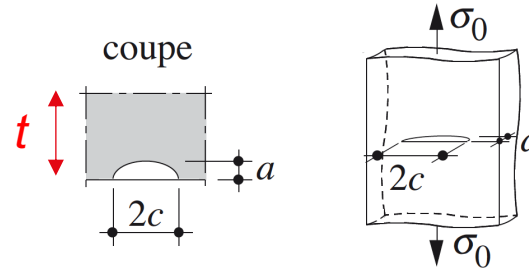
Correction for a surface crack



$$Y_s = 1.12 \text{ pour } \frac{a}{w} \leq 0.05$$

$$Y_s = 1.12 - 0.23 \frac{a}{w} + 10.6 \left(\frac{a}{w} \right)^2 - 21.7 \left(\frac{a}{w} \right)^3 + 30.4 \left(\frac{a}{w} \right)^4 \text{ pour } \frac{a}{w} < 0.45$$

Correction for a surface crack

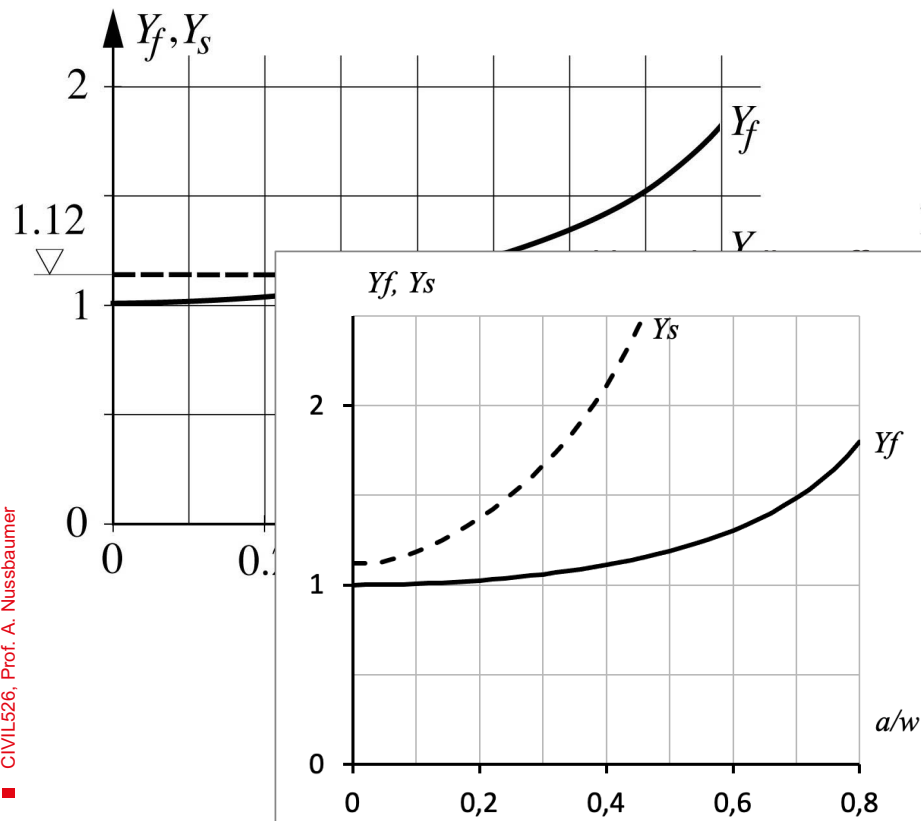


$$Y_s = 1 + 0.12 \left(1 - 0.75 \frac{a}{c} \right)$$

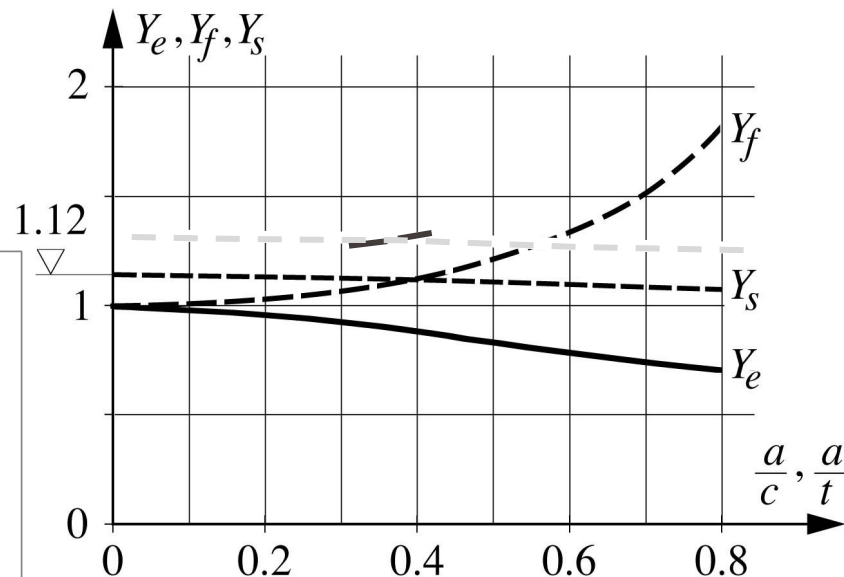
$$a < 0.1 \cdot t$$

Y_s : Solutions for small cracks are only valid when secondary bending effects caused by cracking can be neglected

Two-dimensional case



Three-dimensional case

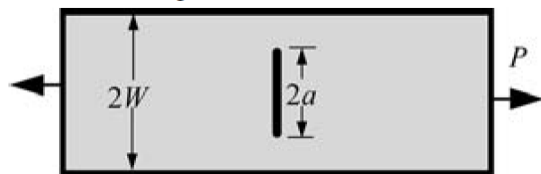


K for standard specimens (Anderson's book)

GEOMETRY

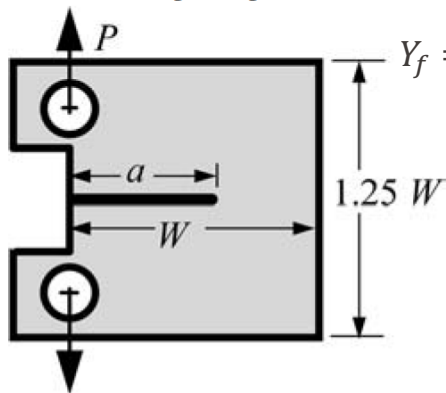
$Y F(a/w) =$

Double Edge Notched Tension (DENT)



$$\frac{\sqrt{\frac{\pi a}{2W}}}{\sqrt{1 - \frac{a}{W}}} \left[1.122 - 0.561 \left(\frac{a}{W} \right) - 0.205 \left(\frac{a}{W} \right)^2 + 0.471 \left(\frac{a}{W} \right)^3 + 0.190 \left(\frac{a}{W} \right)^4 \right]$$

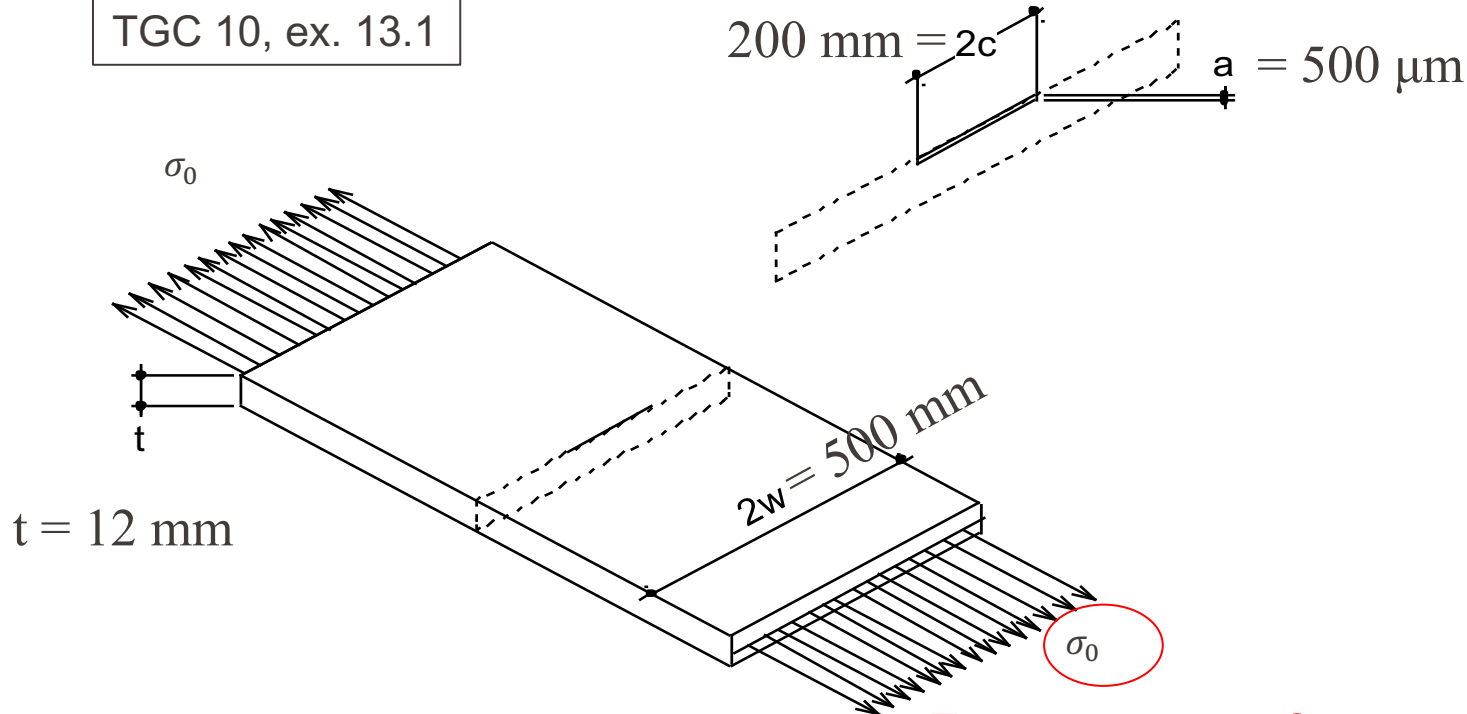
Compact Specimen



$$Y_f = f \left(\frac{a}{W} \right) = \frac{2 + \frac{a}{W}}{\left(1 - \frac{a}{W} \right)^{3/2}} \left[0.886 + 4.64 \left(\frac{a}{W} \right) - 13.32 \left(\frac{a}{W} \right)^2 + 14.72 \left(\frac{a}{W} \right)^3 - 5.60 \left(\frac{a}{W} \right)^4 \right]$$

* $K_I = \frac{P}{B\sqrt{W}} f \left(\frac{a}{W} \right)$ where B is the specimen thickness.

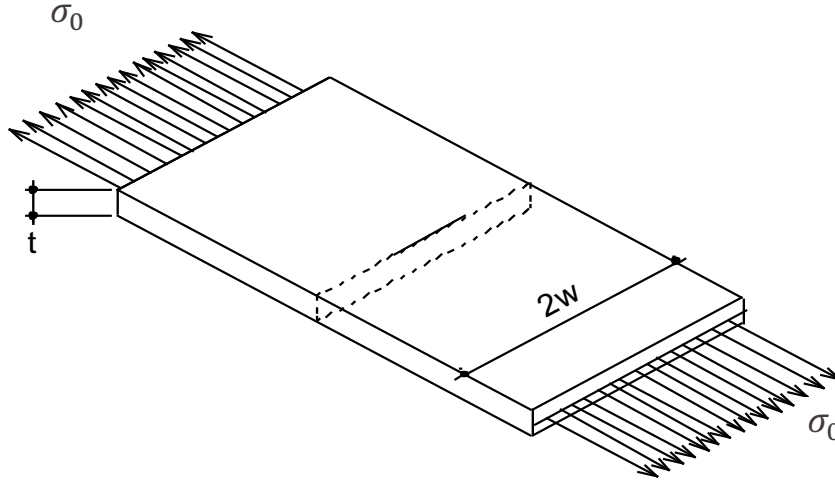
TGC 10, ex. 13.1



Toughness highly depends on material microstructure

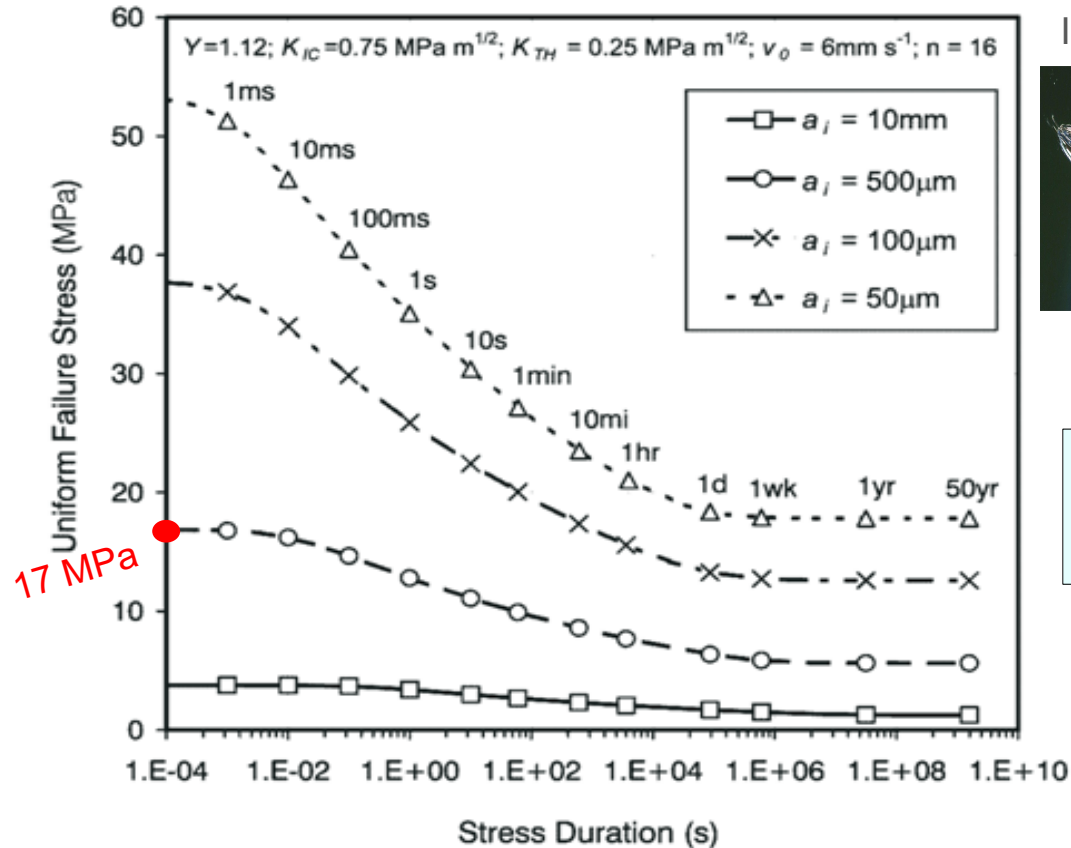
Fracture stress?

- Glass plate, $K_{Ic} = 24 \text{ N/mm}^{3/2}$
- Steel plate, $K_{Ic} = 75 \text{ MPa}\cdot\text{m}^{1/2}$

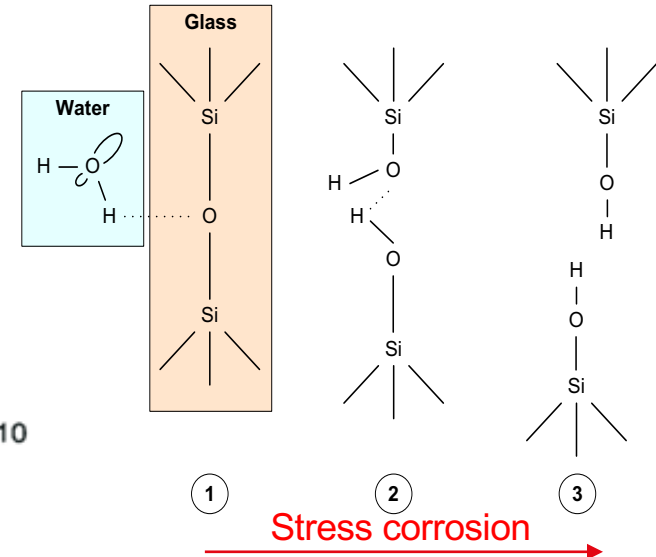


Content

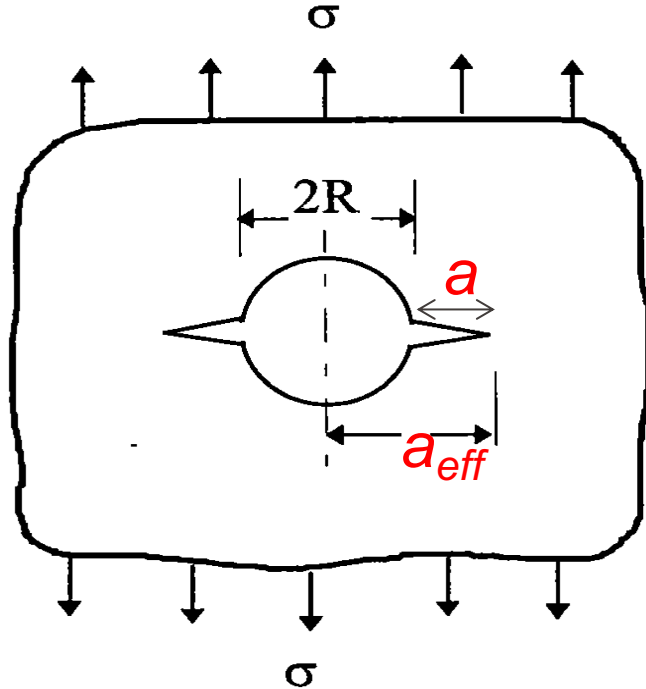
- Introduction
- Stress Intensity Factor
- Influence of shape, correction factors
- **Other influences:**
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- Toughness $\Leftrightarrow$ Charpy V



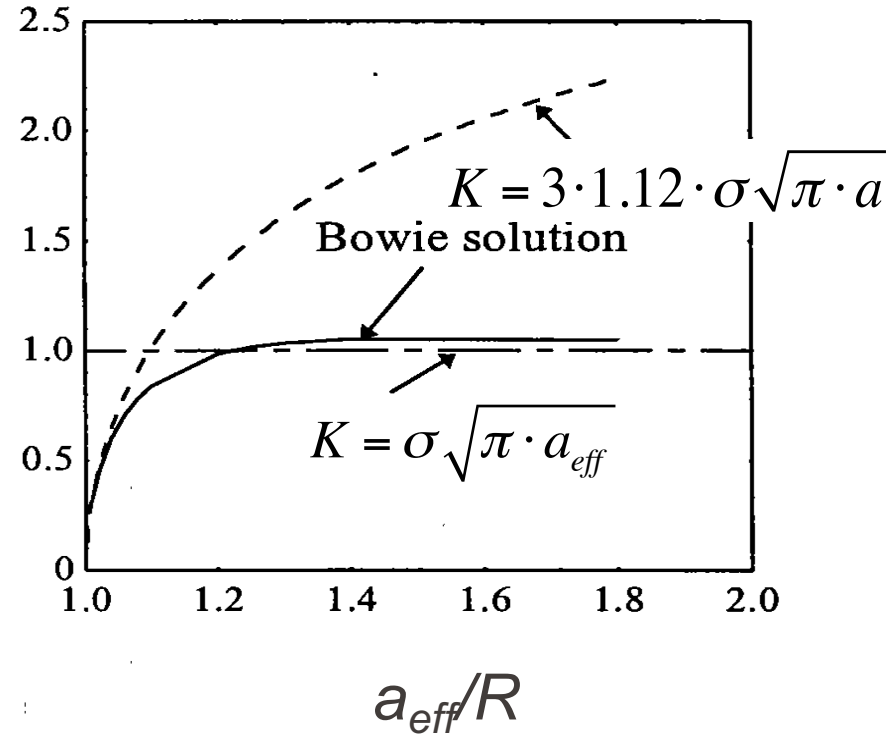
Imperfection at SURFACE



EPFL Comparison of K for a plate with a crack from a hole



Y

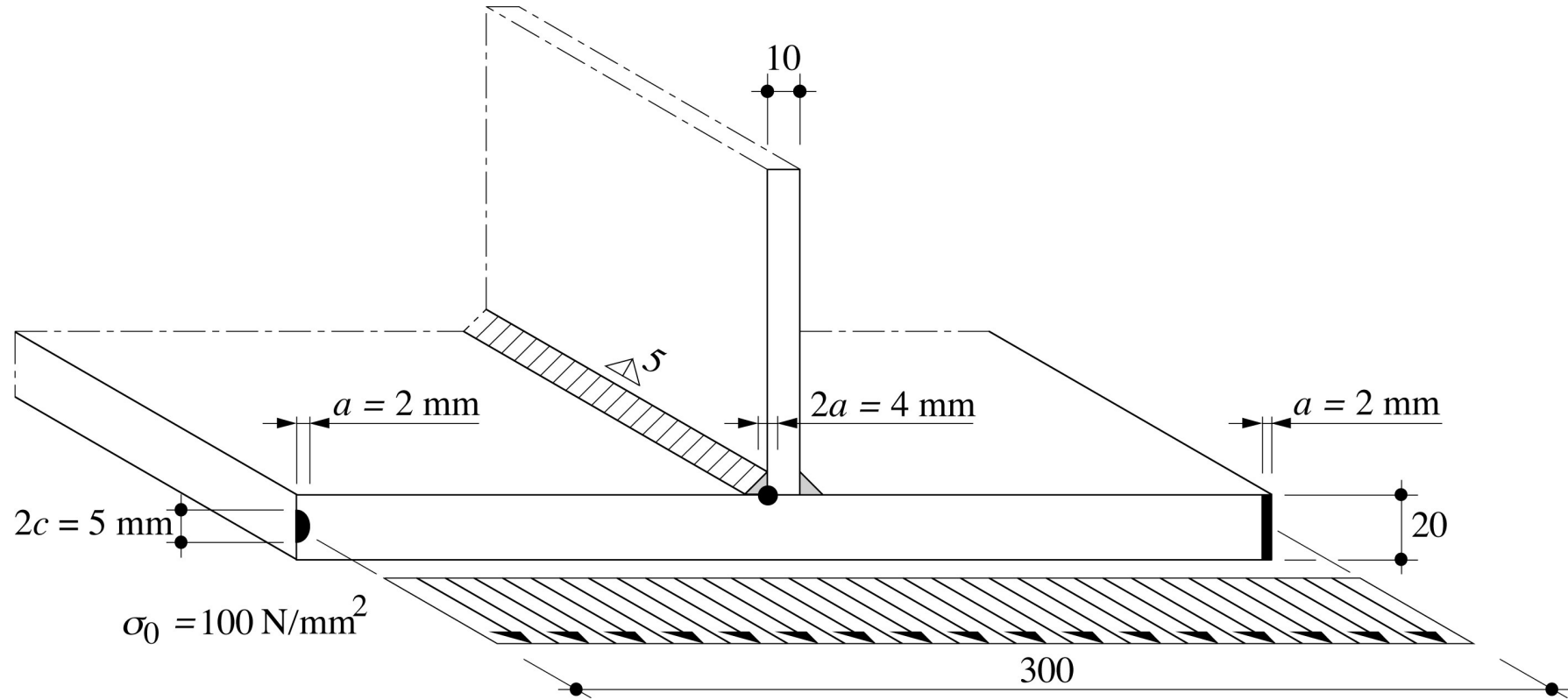


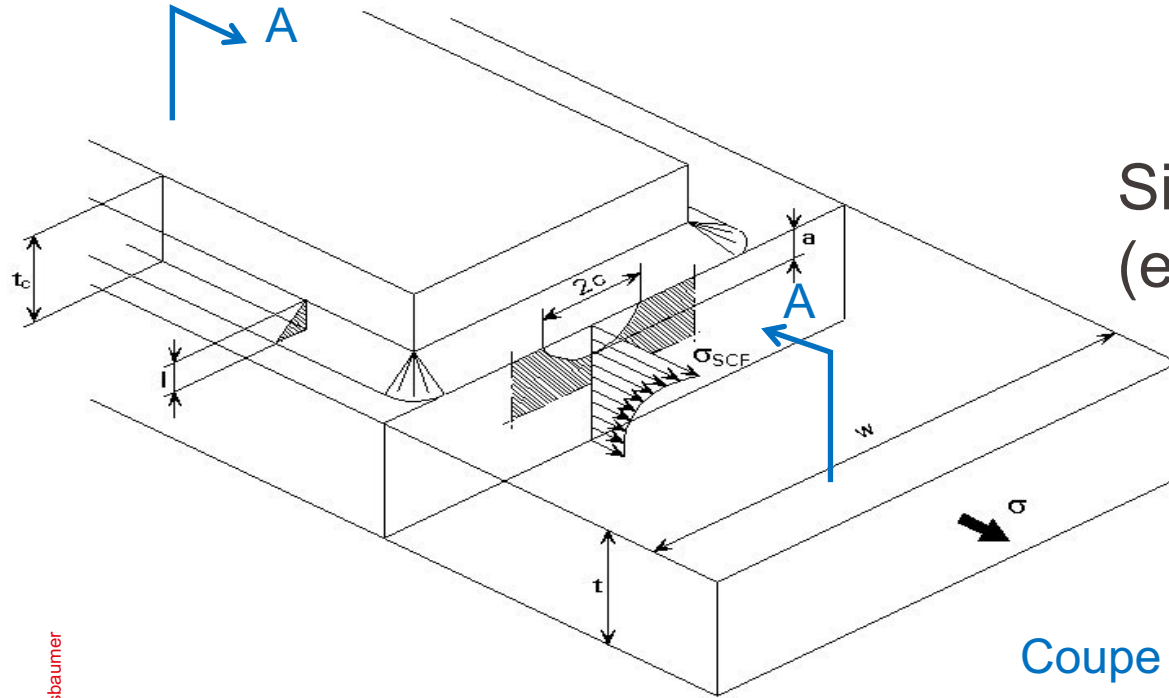
Bowie solution:

<https://engineeringlibrary.org/reference/fracture-mechanics-stress-intensity-factor>

EPFL TGC 10, numerical examples of K calculation (Fig. 13.8)

29



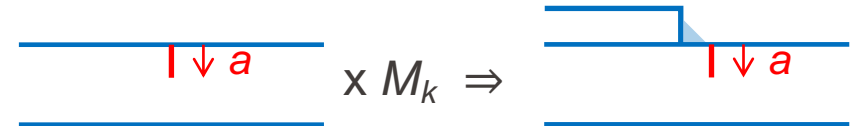


$$K_I = M_k \cdot Y \cdot \sigma \sqrt{\pi a}$$

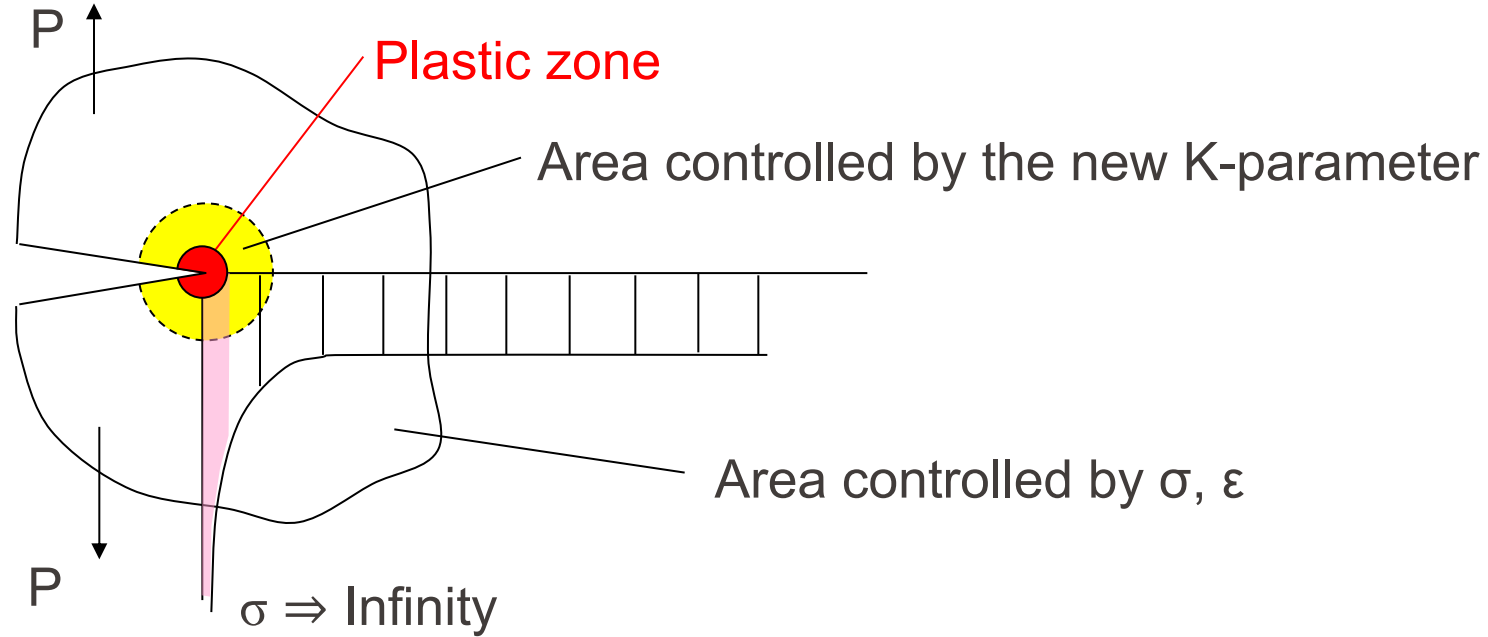
Simple generic solution:
(e.g. Hobbacher)

$$M_k = v \left(\frac{a}{t} \right)^w$$

Coupe A-A



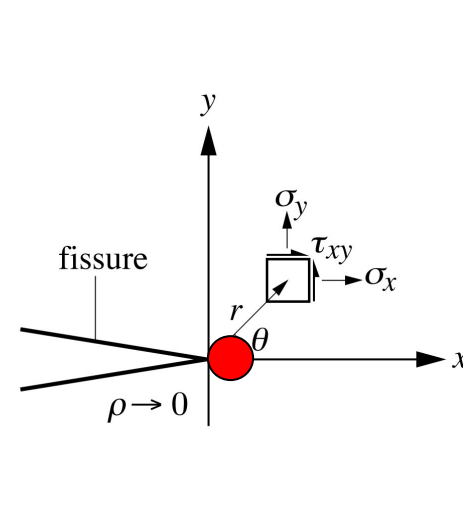
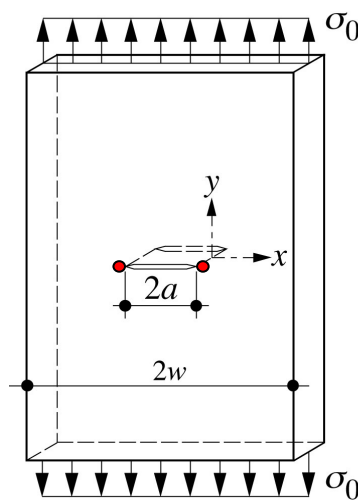
Réf. : Hobbacher, A., Recommendations for Fatigue Design
of Welded Joints and Components, Springer, 2016
<http://link.springer.com/10.1007/978-3-319-19198-0>



Only a 2D representation !!

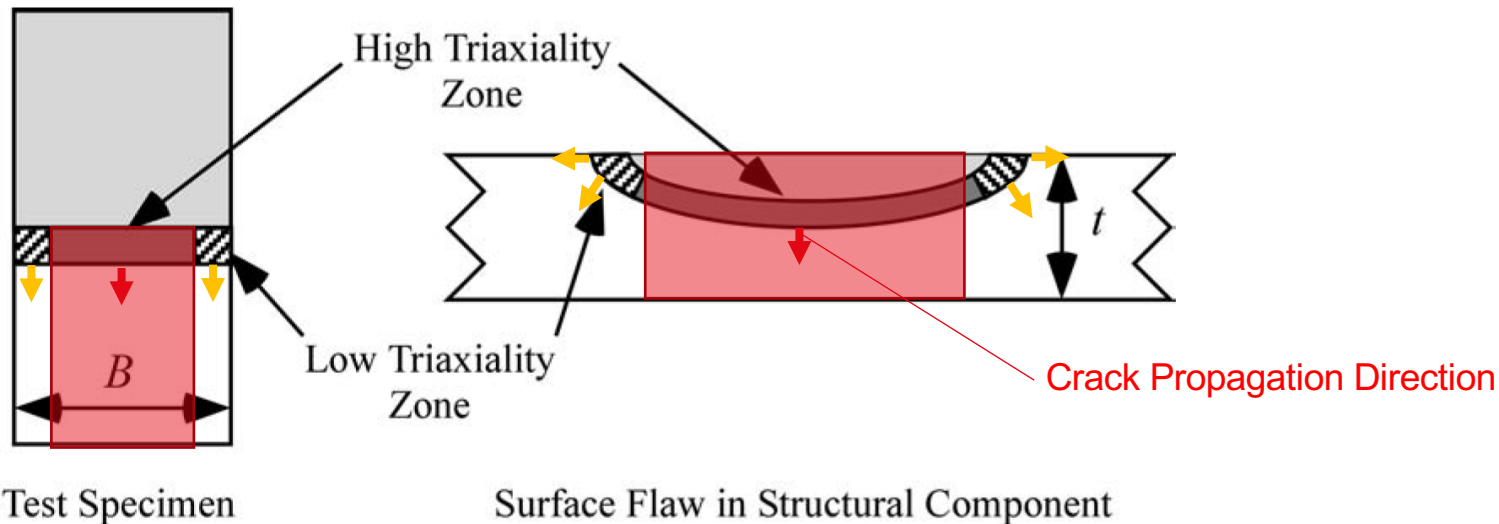
Conditions for K to be a similarity parameter

- 1) The size of the plastic zone (or FPZ=Fracture Process Zone) is small compared to crack size and the cracked body



- 2) The out-of-plane stress states (triaxiality) at crack tip are identical
 $\Rightarrow$ Conditions needed: **plane strain state**
 $\Rightarrow$ Approximative model of reality, generally **conservative**

Containment – different states of triaxiality



- The length-to-diameter ratio of the roll is important in defining the stress state along the crack front
- (Empirical) condition for having the **most critical case = plane strain state**:

$$\frac{B}{r_p} \propto \frac{B}{\left(K_I / f_y\right)^2} \geq 2.5$$

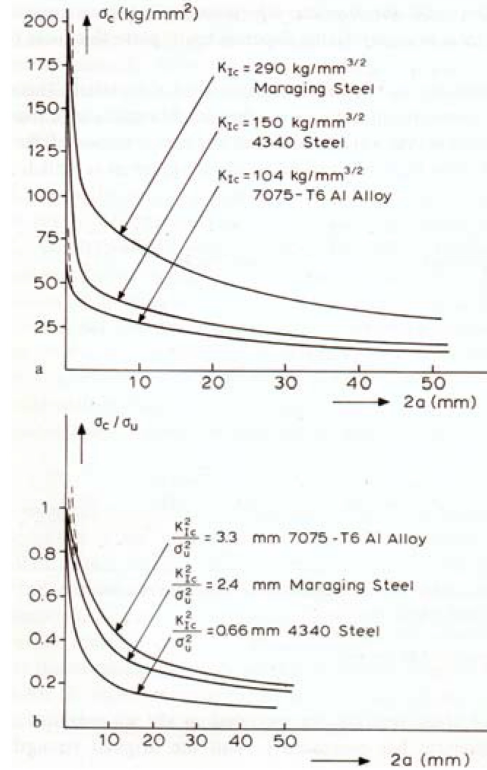
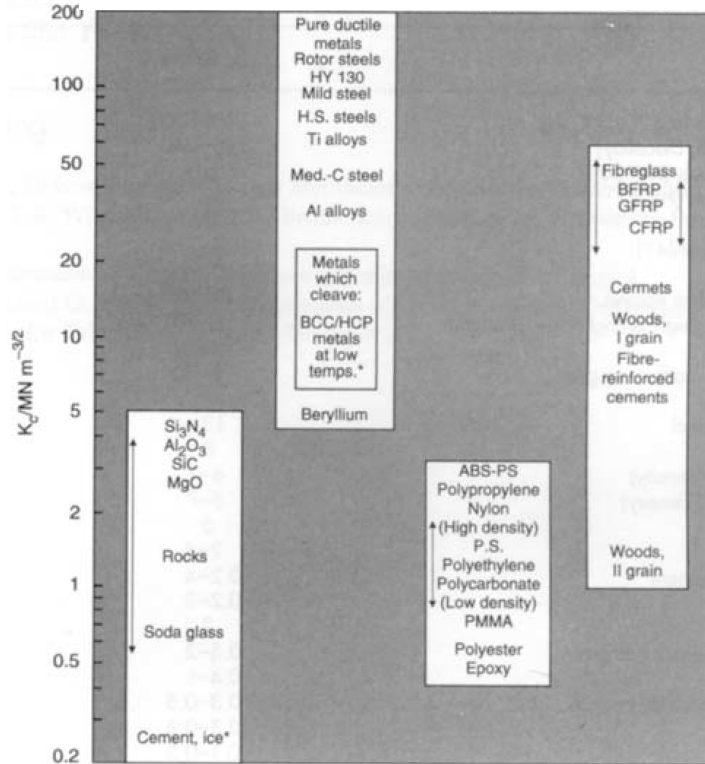
Content

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- **Toughness $\Leftrightarrow$ Charpy V**

It depends on the microstructure, the triaxiality, and external variables

Ceramics Metals Polymers Composites

Temperature (and environment)
(degradation)
Loading rate

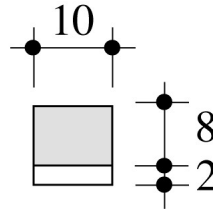
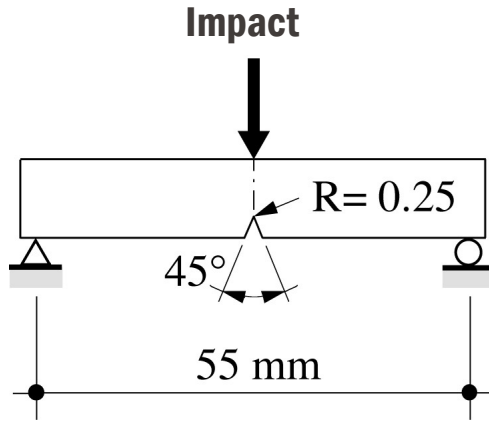


Typical toughness values K_{Ic} (plane strain state, at ambient temperature, quasi-static loading)

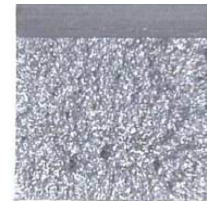
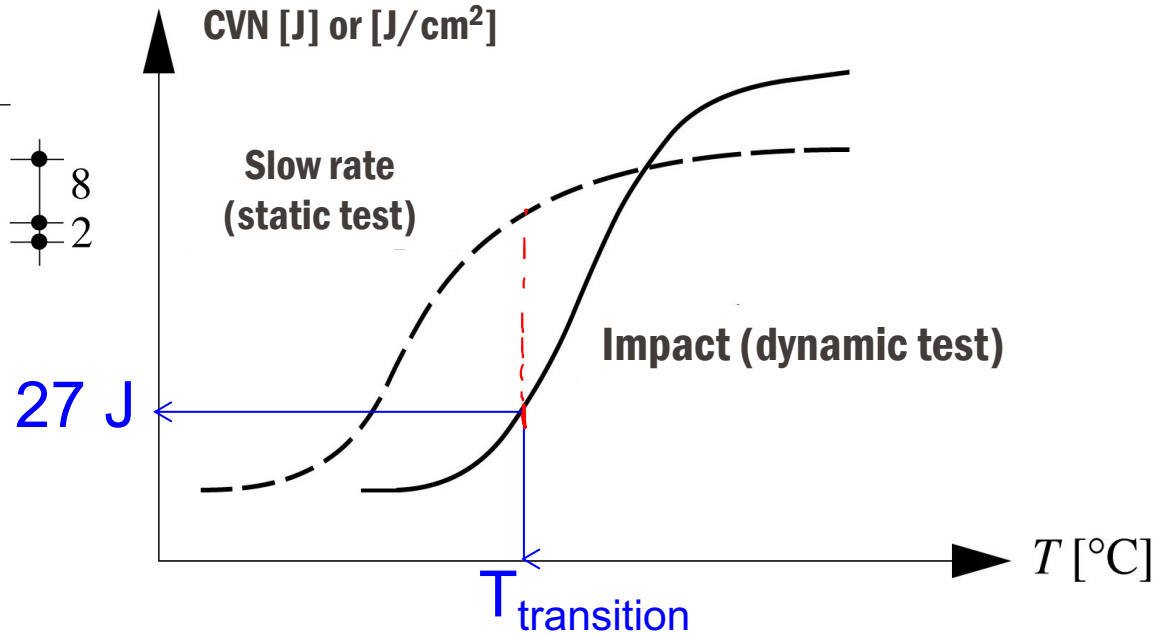
Matériau	Particularités	Limite élastique f_y, f_t	Ténacité, K_{Ic}		Epaisseur minimale requise B
		[N/mm ²]	[N/mm ^{3/2}]	[ksi√in]	[mm]
<i>Acier</i>					
S 235	acier carbone	235	> 5000	> 140	1100
S 355	acier carbone	355	> 5000	> 140	500
4340	acier durci	1810	1470	43	1.7
<i>Titane</i>					
6Al-4V	($\alpha+\beta$)STA	1100	1200	35	3
13V-11Cr-3Al	STA	1130	870	25	1.5
<i>Aluminium</i>					
7075	T651	540	920	27	7.3
2014	T4	450	880	26	9.6
2014	T3	390	1080	31	19
<i>Non-métalliques</i>					
PVC	chlorure de vinyle	45	60-250	2-8	77
Résine époxyde		30-100	16	0.5	-
Polyméthyl méthacrylate	Plexiglas		22 to 50	0.6 to 1.5	-
Verre ordinaire	Soda-lime glass	3600	22	0.7	-
Bétons	C12/15 to C50/60	1.6 to 4	7 to 44	0.2 to 1.3	-
Béton (selon doc. Brühwiler)		3.6	58.8	1.7	-

Note : ksi√in $\approx$ MPa√m

TGC 10, figs. 3.19 and 3.20: for steels, Charpy impact test (or Charpy V notch test)



Note: test invented by G.R. Irwin



EPFL Relationship CVN - Toughness

For a first estimation,
deduction from Charpy V
tests (for C-Mn steels):

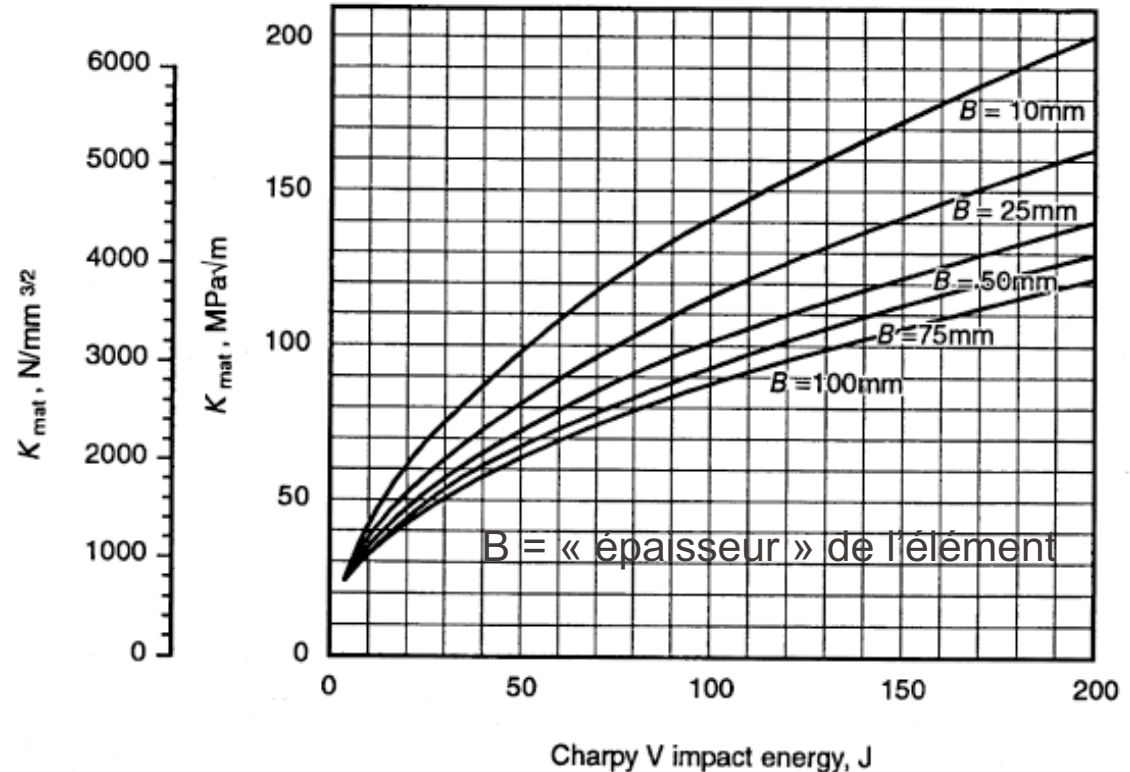


Figure J.2 — K_{mat} plotted against Charpy V impact energy for lower shelf and transitional behaviour

(tiré de BS PD7910, 2005)

Reminder: Example of a material certificate



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CERTIFICATO DI COLLAUDO / INSPECTION CERTIFICATE (EN 10204/92 3.1B)

CLIENTE / CUSTOMER : LUDWIG STAHL AG
 INDIRIZZO / ADDRESS : TWS GEWERBEZENTRUM POSTFACH
 CAP. CITTA' / TOWN : 8370 SIRMACH CH
 N° ORDINE / ORDER No : 105582
 Na. ORDINE / MILL ORDER : 484633
 N° CERTIFICATO / CERTIFICATE No : 013270
 Data: 27/02/02
 PRODOTTO : LAMIERE IN ACCIAIO LAMINATE A CALDO - HOT ROLLED STEEL PLATES
 STANDARD : EN 10025, EN 10029

QUALITA' QUALITY	ID.LAMIERA ID.PLATE	DIMENSIONI DIMENSIONS			Fogli Plates	N° COLATA HEAT No	ANALISI CHIMICA CHEMICAL COMPOSITION										CARATTERISTICHE MECCANICHE MECHANICAL TEST				
							C %	Mn %	Si %	S %	P %	Cr %	Ni %	Cu %	Al %	Mo %	CEQ %	Rm TS	ReH YP	A Elong.	Kv -20 °C J
S355J2G3	36972	15	2000	8000	7	1107372	0,16	1,49	0,32	0,009	0,020	0,06	0,02	0,03	0,040			529	384	26,0	51
S355J2G3	36408	25	2000	8000	4	215157	0,20	1,45	0,21	0,015	0,027	0,030	0,010	0,010	0,033			542	389	28,5	47
S355J2G3	36849	30	2000	8000	1	315275	0,20	1,36	0,21	0,018	0,025	0,050	0,020	0,200	0,023			537	356	27,0	40
S355J2G3	36980	30	2000	8000	1	215182	0,21	1,39	0,23	0,011	0,022	0,040	0,020	0,020	0,043			540	359	27,0	40

DEBRUNNER TAVELLI SA

SI CERTIFICA CHE IL PRODOTTO SOPRAELENCATO E' CONFORME ALLE PRESCRIZIONI DELL'ORDINE.
 WE HEREBY CERTIFY THAT THE ABOVE MENTIONED MATERIAL IS CONFORM TO THE QUALITY SPECIFIED.

SIA 263, Appendix A: Maximum permissible thicknesses

Railway steel bridge S 355, $t_{\max} = 75 \text{ mm}$, $T_{\min} < -10 \text{ °C} \Leftrightarrow \text{S355 K2}$

Température de service déterminante de l'élément de construction		$T_{\min} \geq 0 \text{ °C}$		$0 \text{ °C} > T_{\min} \geq -10 \text{ °C}$		$T_{\min} < -10 \text{ °C}$	
Catégories de service		SC1	SC2	SC1	SC2	SC1	SC2
Nuances d'aciers	S 235 JR	$t \leq 150$	$t \leq 40$	$t \leq 100$	$t \leq 30$	$t \leq 40$	–
	S 235 J0	250	100	250	85	180	60
	S 235 J2	250	140	250	120	250	85
	S 355 JR	40	–	30	–	20	–
	S 355 J0	120	80	80	65	60	45
	S 355 J2	250	110	180	90	150	65
	S 355 K2	250	120	250	100	250	80
	S 355 N	250	130	250	110	250	90
	S 355 NL	250	175	250	150	250	130
	S 460 N	180	50	150	40	100	30
	S 460 NL	250	150	250	100	250	70

Table similar to those of EN 1993-1-10: choice of quality for toughness

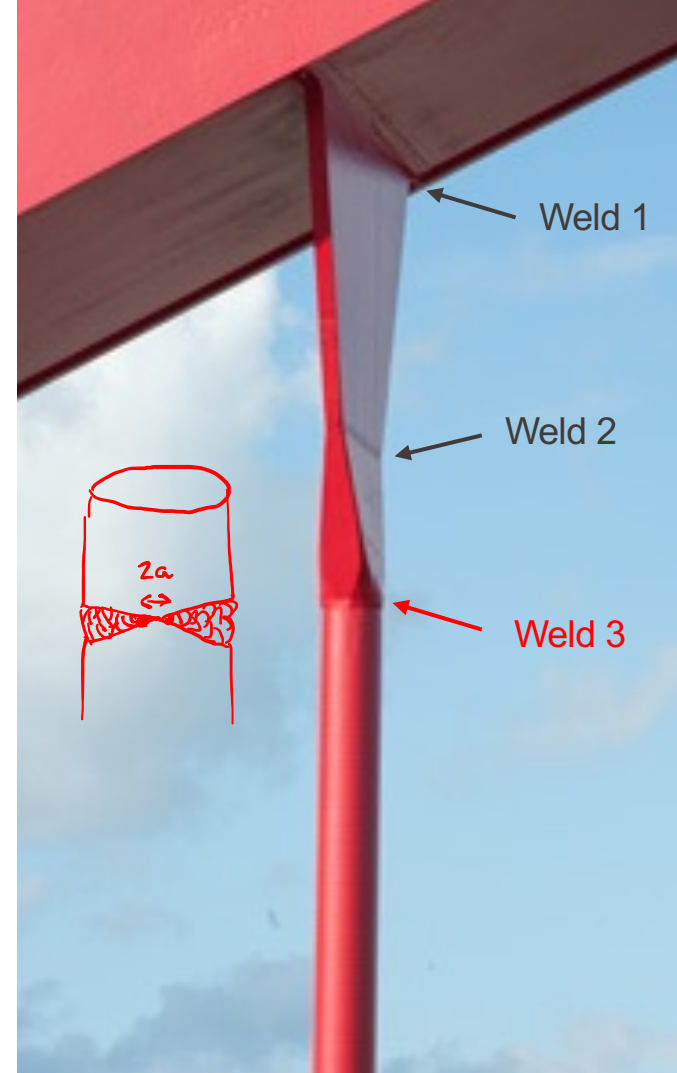
EXERCISE FAT3: Stress Intensity Factor Calculations

- Arch bridge hanger: 100mm S690 steel **round bar**
- Welded (full penetration) at its ends: connecting piece then plate
- **Weld 3 to be checked**
- Measured CVN (valid down to -60°C)
135 J = guaranteed min. value

Questions:

1. Max. circular internal imperfection size?
Bar designed using $\sigma_{Ed} = 80\% \cdot f_y$
2. Semi-elliptic, imperfection $a/c = 0.5$,
 $2c = 30$ mm critical $\sigma_{Ed} = 1.1 \cdot 80\% \cdot f_y$?

Note: no additional partial factors.



The background of the slide is a black and white photograph showing several CT (Compact Tension) specimens of different sizes. They are arranged in a stack, with some being held by a mechanical testing machine. The specimens are made of a metallic material, likely steel, and have a central hole. The testing machine has a diamond-patterned safety cage.

Eprouvettes CT de différentes tailles (acier A533B,
 $f_y = 480$ MPa), réf.: Barsom&Rolfe, 1987

**Thank
you**

**Prof. A. Nussbaumer
RESSLab**

EXERCICE FAT3:

Sketch of solution

Cross-sectional view of hanger bar:

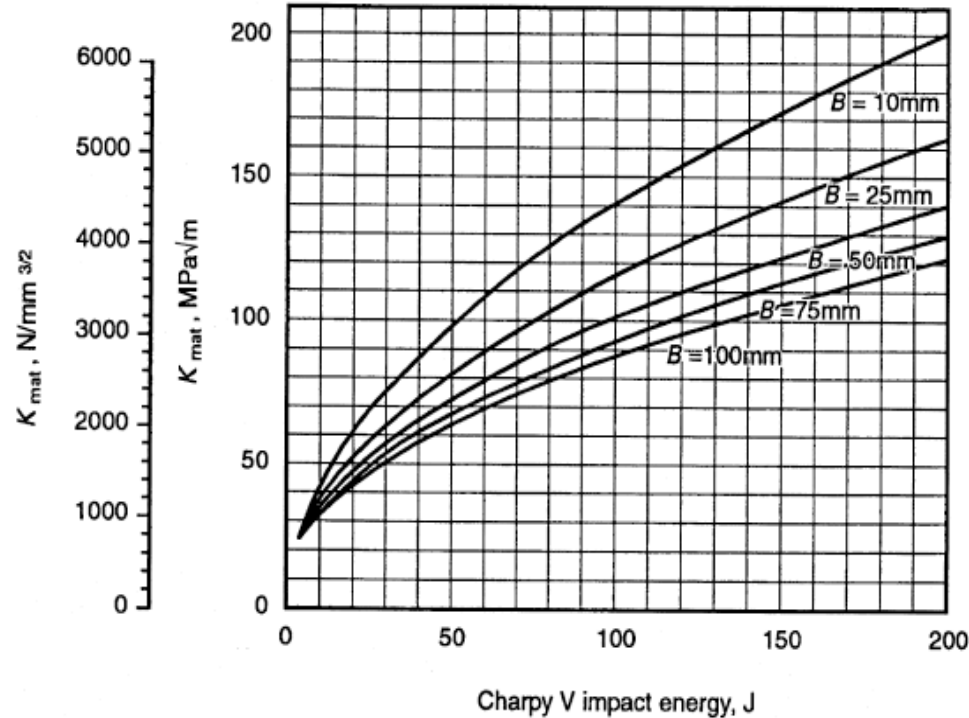
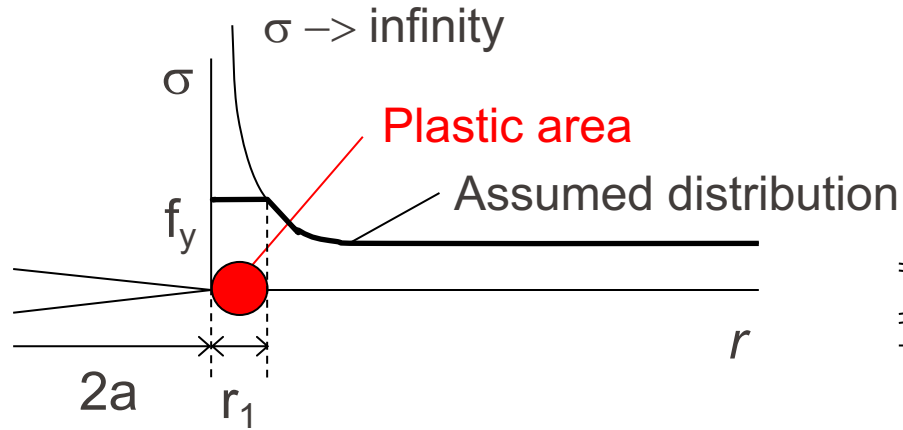


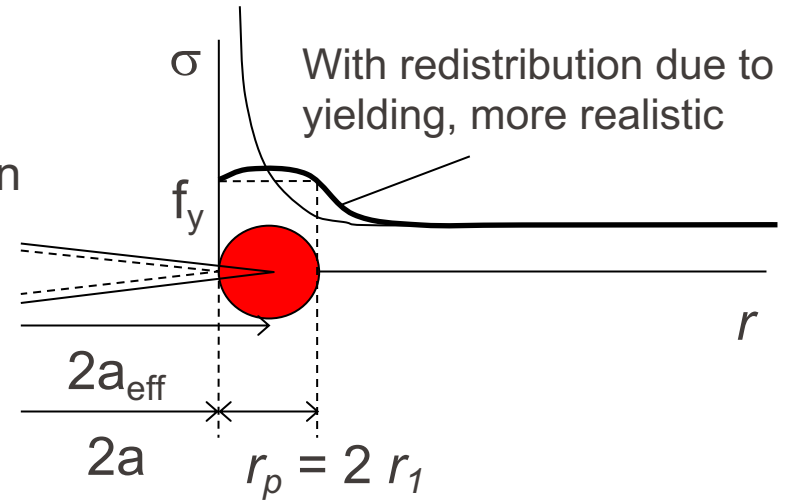
Figure J.2 — K_{mat} plotted against Charpy V impact energy for lower shelf and transitional behaviour



$$\sigma_{yy} = \frac{K_I}{\sqrt{2\pi r_1}} f_{ij}(\theta = 0) = f_y$$

$$r_1 = \frac{K_I^2}{2\pi f_y^2}$$

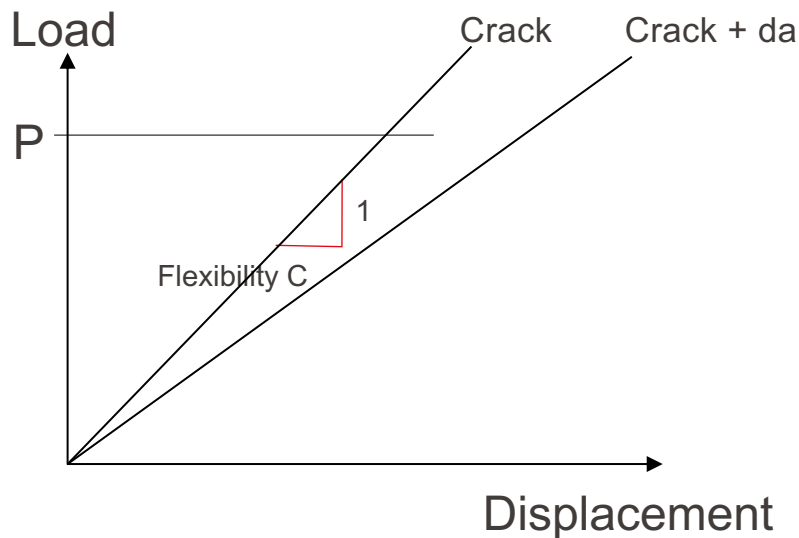
$\sigma_z = 0$ Plane stress state



$$r_1 = \frac{K_I^2}{6\pi f_y^2}$$

$\varepsilon_z = 0$ Plane strain state

- Parameter G is a direct measure of the material's ability to support the plastic area, to resist, in front of the crack peak until it propagates.



- Under disp. or cst load :

$$G = \frac{1}{2} P^2 \frac{dC}{dA}$$

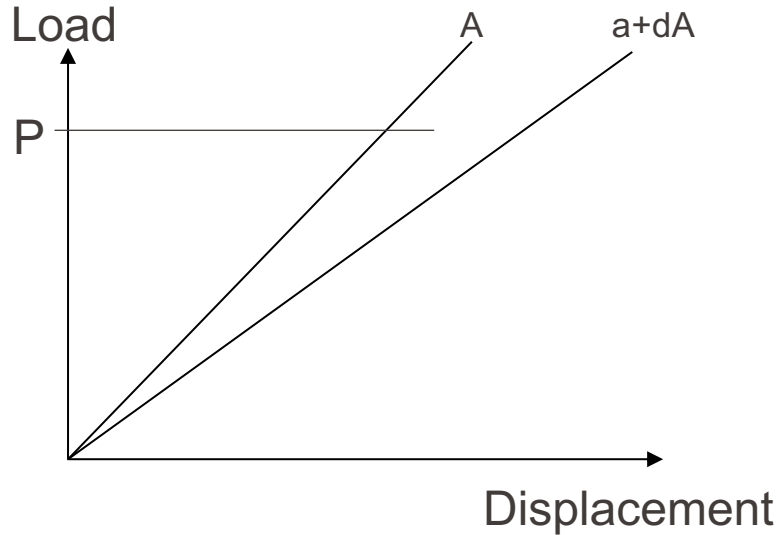
- Link to K:

$$G = \alpha \frac{K_I^2}{E}$$

$$\alpha = 1 \quad \text{Plane Stress State}$$

$$\alpha = 1 - \nu^2 \quad \text{Plane Strain State}$$

Irwin theory, crack driving force concept



- Under constant load:

$$G_c = \frac{1}{2} P^2 \frac{dC}{dA} = G_{\Delta}$$

Under constant displacement

- The proof that this parameter is a direct measure of material capacity to withstand a plastic zone, to resist, in front of crack tip

- Modified Griffith energy criterion, add to it plastic dissipation:

$$\sigma \sqrt{a} \geq \sqrt{\frac{E \cdot G}{\pi}}$$

$$G = 2\gamma + G_{Plast}$$